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VEHICLE DYNAMICS SIMULATION, PART 1: MATHEMATICAL MODEL

Iztok Ciglaric and Ivan Prebil

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1. Introduction

The method of multibody systems [Melzer, 1996] is well established and accepted tool in the dynamic analysis of mechanisms, machines, vehicles etc. Computer based assembly of the kinematics and kinetic equations of motion in symbolic form for a multibody system is an important tool in modern dynamic analysis. The symbolic generation of the equations of motion is particulariy attractive since the equations have to be generated only once and afterwards numerically analyzed for various parameter values. Also the classical engineering analysis could be easily extended to perform design sensitivity analysis or stability analysis. Nevertheless, there are limitations in symbolic computing, in particular if the multibody system is complex, with many degrees of freedom. Drawbacks in this case are that for complex multibody system a long and tedious symbolic expressions are generated, which may be difficult to handle by computer algebra systems such as *MATHEMATICA* [Mathematica, 1996], and which may be numerical ineffective. Therefore it is necessary to formulate such multibody system formalism that is capable to circumvent such problems.

2. Multibody system formalism

Multibody system is defined as a set of rigid and flexible bodies connected by joints and massless force elements. Joints are lower and higher kinematic pairs that characterize relative motion between bodies. Force elements are springs, dampers and others energy transformation elements such as Coulomb damping element. An example multibody system is depicted in fig. 1.

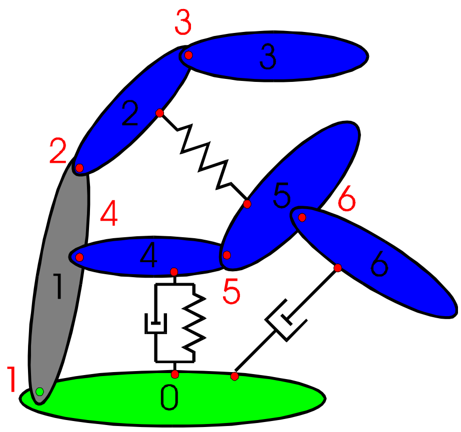

Figure 1. Multibody system

In order to obtain mathematical model of multibody system it is necessary to make suitable mathematical formulations of bodies, joints and topology of the structure as a whole.

2.1 Body description

Body is a fundamental element of multibody system. A body N_i (rigid or flexible) in the multibody system is linked to inertial body through the parent body by joint *i (inboard joint)*, and can have several children bodies N_j attached through joints *j (outboard joints)*. From mechanical point of view body N_i can be defined with the following attributes:

- Reference attachment point Oi of inboard joint i on body Ni.
- Reference attachment points Oj of outboard joints j on body Ni.
- The center of mass Gi of bodies Ni.
- A moving coordinate frame $(O^i, \vec{e}_1^{\,i}, \vec{e}_2^{\,i}, \vec{e}_3^{\,i})$ fixed to body Ni and located at the point Oi.
- The position vectors $\vec{r}_G^{\,i}$ of the centre of mass Gi with respect to coordinate frame $(O^i, \vec{e}_1^{\,i}, \vec{e}_2^{\,i}, \vec{e}_3^{\,i})$.
- The position vector $\vec{r}_j^{\,i}$ of the outboard joints j with respect to coordinate frame $(O^i, \vec{e}_1^{\,i}, \vec{e}_2^{\,i}, \vec{e}_3^{\,i})$.
- Mass mi and inertia tensor Ji with respect to center of mass Gi.

One has to note that beside these basic attributes, flexible bodies need additional description that will be described in detail in the next chapter.

2.2 Flexible body description

Superelement approach and the so-called continuum approach are special multibody techniques for incorporating flexible bodies into multibody system. The former describes flexible body as a series of rigid bodies interconnected by elastic force elements. The latter is used in this research and describes the flexible body in terms of shape functions. In this case the kinematics of a flexible body is expressed as superposition of the reference configuration body motion (gross motion of rigid body) and displacement due to deformation, measured with respect to the body reference configuration [Valembois, 1997]. The position of an elemental mass dm_i located on the body i can be described with respect to the inertial reference frame as

$$\mathbf{R}_{dm_i} = \mathbf{D}_i + \mathbf{T}_i\left[\vec{r}_i + \vec{g}_i[\vec{r}_i]\right], \tag{1}$$

where $\mathbf{D}_i$ are the position vectors and $\mathbf{T}_i$ rotation matrixes. Vectors $\vec{r}_i$ describe the displacement of dm_i due to rigid body motion and $\vec{g}_i[\vec{r}_i]$ represent the displacement due to deformation of the body.

Deformation of the i-th body is expressed as the product of varying admissible shape functions $f^i[\vec{r}_i]$ and weighting coefficient

$$\vec{g}_i[\vec{r}_i] = \sum_{i=1}^{N} f^i[\vec{r}_i] d_i . \tag{2}$$

This research focuses on elastic transverse deformation of Euler-Bernoulli beams. In this case the admissible shape function of *k-th* mode is [Caron, 1998]

$$f_{yk}^{i}(x_i, l_i) = c_{ik1}\cosh\left(\frac{l_{ik}x_i}{l_i}\right) + c_{ik2}\sinh\left(\frac{l_{ik}x_i}{l_i}\right) + c_{ik3}\sin\left(\frac{l_{ik}x_i}{l_i}\right) + c_{ik4}\cos\left(\frac{l_{ik}x_i}{l_i}\right) \tag{3}$$

where l_i are the length of the *i-th* body (beam) and x_i the position along l_i. Coefficients c_{ikj} and l_{ik} depend on boundary conditions of considered elastic body modeled as an Euler-Bernoulli beam.

2.3 Joint description

Joints link the bodies and restrict relative motion between them. The configuration space G of relative motion between two reference coordinate frames fixed on two bodies is Special Euclidean group consisting of all rotations and translations in R^3 space. An element of this Special Euclidean group may be represented by the transformation matrix [Koivo, 1989]. In general a relation on the tangent bundle $\mathbf{T}G$ characterizes the joint. Such a relation is usually expressed in local coordinates as

$$f(\vec{q},\dot{\vec{q}}) = 0 \tag{4}$$

where $f : \mathbf{T}G \to R^k$. Natural constraints almost always occur in the following two forms

$$f(\vec{q}) = 0 \tag{5a}$$

or

$$F(\vec{q})\dot{\vec{q}} = 0 \tag{5b}$$

Note that constraints of form (5a) define submanifold of G [Wasserman, 1992] and restrict the relative position of two bodies. Constraints of this form are often called geometric constraints. Constraints of the form (5b) restrict the relative velocity of two bodies and they are often called kinematic constraints. More general and more convenient characterization of equation (5b) is

$$\mathbf{A}(\vec{q})\,\vec{p} = 0 \tag{6}$$

where $\mathbf{A}$ is linear operator that could be represented in terms of matrix and $\vec{p} \in Ker[\mathbf{A}(\vec{q})]$. In the case of simple kinematic joints where the motion axes are fixed in at least one of the bodies the operator $\mathbf{A}$ is independent of configuration and therefore constant. In this case the solutions of (6) are of the form

$$\vec{p} = \mathbf{H}\vec{q} \tag{7}$$

where $\mathbf{H}$ is called the joint map matrix. It is of full rank $r = \dim Ker[\mathbf{A}]$ and the equality $Ker[\mathbf{A}] = \mathrm{Im}[\mathbf{H}]$ is valid. The columns of $\mathbf{H}$ form a basis for $Ker[\mathbf{A}]$.

In the case when it is possible to define the action of a joint in terms of a sequence of simple kinematic joints, we call such joints compound kinematic joints. In general, a compound joint is defined as a joint that can be characterized as the relative motion of a sequence of reference frames such that relative motion between two successive frames is defined by a simple kinematic joint. Every simple joint is characterized by a joint map matrix H^i, i=1,2,3,…,n

2.4 Topology description

As far as the topology is concerned the following conventions are adopted (see fig. 1):

- Bodies are numbered in ascending order from the inertial body (index 0) to the terminal bodies.
- A joint, which precedes a body, has the same index as the body itself.

To establish the equation of motion the topological configuration of multibody system has to be represented in mathematical terminology. To represent the topological structure of the multibody system the pertinent information is mapped onto a graph. Each body is put into correspondence with a vertex of a graph. The vertices are enumerated $i=1,2,...,N_v$. A vertex labeled $i=0$ is adjoined to the reference inertial body. Vertices are interconnected by single branches, representing the interactions of bodies upon each other. The branches are enumerated $j=1,2,...,N_b$ It should be noted that each branch could represent more than one physical link that connects two bodies (such as joints, springs, dampers etc.) To give a graph a mathematical characterization we define an incident matrix in terms of vertices that are adjacent to one another or in terms of vertices on which an edge or branch is incident

$$\mathbf{S} = \left[s^{ij}\right], \quad i = 1,2,...,N_v, \quad j = 1,2,...,N_b \tag{8}$$

In column j the value of +1 is entered at the row number corresponding to the vertex from which edge j emanates. The value –1 is entered at the row number corresponding to the vertex on which edge j terminates. The reminder of the column is filled with the element 0. This is done for all columns.

2.5 Mathematical model-the equations of motion

The equations of motion can be obtained by using Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\bar{q}}} - \frac{\partial L}{\partial \bar{q}} = Q \text{, where } L(\bar{q},\dot{\bar{q}}) = E_{kin}(\bar{q},\dot{\bar{q}}) - E_{pot}(\bar{q}) \tag{9}$$

In (9) $E_{kin}(\bar{q},\dot{\bar{q}})$ is kinetic energy and $E_{pot}(\bar{q})$ is potential energy of the multibody system. In the case of multibody system it is easier to formulate the equations of motion in terms of velocity variables which can not be expressed as the time derivatives of any corresponding configuration coordinate from vector $\bar{q}$. If $\mathbf{M}$ is the m-dimensional configuration manifold [Wasserman, 1992] for Lagrangian system (9), and $\bar{q}(t):[t_1,t_2] \rightarrow \mathbf{M}$ is a smooth path then $\dot{\bar{q}}(t)$ is the tangent vector to the path at the point $\bar{q}(t) \in \mathbf{M}$. In this case it is always possible to express $\dot{\bar{q}}(t)$ as linear combination of tangent vectors

$$\dot{\bar{q}} = \mathbf{V}(\bar{q})\bar{p} \tag{10}$$

Using (10) and set equality $\hat{L}(\bar{q},\bar{p}) = L(\bar{q},\dot{\bar{q}})$ one can express (9) using Hamilton's principles [Jain, 1991] as

$$\frac{d}{dt}\frac{\partial \hat{L}}{\partial \dot{\bar{p}}} - \frac{\partial \hat{L}}{\partial \bar{p}}\mathbf{V}^{-1}\sum_{i=1}^{n} p_i X_i - \frac{\partial \hat{L}}{\partial \bar{q}}\mathbf{V} = Q^T\mathbf{V} \tag{11}$$

where $X_i = [[v_i,v_1],[v_i,v_2],...,[v_i,v_m]]$ and

$$\left[v_i,v_j\right] = \sum_{k=1}^{m} a_{ij}^k(\bar{q})v_k \tag{12}$$

is linear combination of linearly independent vectors $\left[v_1,v_2,...,v_m\right]$ on configuration manifold M. Expression (11) can be reduced to the form

$$\mathbf{M}(\bar{q})\dot{\bar{p}} + \mathbf{C}(\bar{q},\bar{p})\bar{p} + \mathbf{F}(\bar{q}) = \mathbf{Q}_p \tag{13}$$

In (13) M is system inertia matrix, calculated as

$$\mathbf{M} = \mathbf{H}^T\mathbf{F}^T\mathbf{M}_0\mathbf{F}\,\mathbf{H} \tag{14}$$

C and F right hand side of Poincare's equations calculated

$$\mathbf{C}(\bar{q},\bar{p}) = \left[\frac{\partial \mathbf{M}\bar{p}}{\partial \bar{q}}\mathbf{V}\right] + \frac{1}{2}\left[\frac{\partial \mathbf{M}\bar{p}}{\partial \bar{q}}\mathbf{V}\right]^T + \left[\sum_{i=1}^{m} p_i X_i^T\right]\mathbf{V}^T\mathbf{M} \tag{15}$$

$$\mathbf{F}(\bar{q}) = \mathbf{V}^T(\bar{q})\frac{\partial E_{pot}}{\partial \bar{q}^T} \tag{16}$$

and Qp generalized forces represented in the $\bar{p}$ -vector basis calculated as

$$\mathbf{Q}_p = \mathbf{V}^T(\bar{q})\mathbf{Q} \tag{17}$$

$\mathbf{Q}$ denotes the generalized forces in the $\bar{q}$ -vector basis. In (14) $\mathbf{M}_0$ is the spatial inertia matrix and $\mathbf{F}$ is spatial velocity transformation matrix. Note that the matrix $\mathbf{F}$ and product $\mathbf{F}\,\mathbf{H}$ can be recursively computed [Ramakrishan, 1989]. Once system inertia matrix is calculated, one compute $\mathbf{C}(\bar{q},\bar{p})$, $\mathbf{F}(\bar{q})$ and $\mathbf{Q}_p$ using (15), (16) and (17) respectively, assuming that the potential energy function $E_{pot}(\bar{q})$ and the generalized force vector $\mathbf{Q}$ are available.

3. Mechanical and mathematical model of the road vehicle

The proposed multibody system formalism is general and could be used for any kind of mechanical system with arbitrary degree of freedom and complexity. However this general proposed tool has been used to handle vehicle dynamic to demonstrate its efficiency. It is well known that vehicle dynamic is one of the most complex dynamic phenomenon [Crolla, 1995] and therefore suitable benchmark.

3.1 Mechanical model

Fig. 2 shows the proposed mechanical model of the road vehicle. The model has up to 26 DOF. The vehicle chassis (unsprung mass) has 6DOF. Each of four suspensions has 1 DOF that enable wheel vertical displacement.

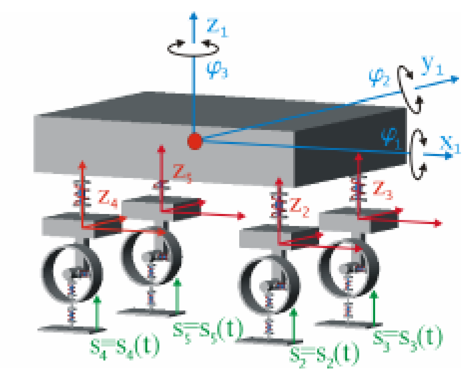

Figure 2. Vehicle mechanical model

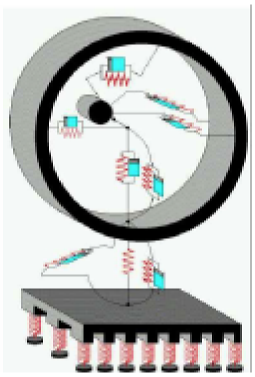

Figure 3. Virtual tyre combined slip model

Suspension could be modeled as lumped mass model (fig. 4), or swing arm model (fig. 5). For the lumped mass model the suspension components form single mass (sprung mass). Mass is connected to the vehicle chassis at the wheel centre by translation kinematic pair which allows vertical sliding motion of sprung mass. Swing arm model also represents single mass, connected to the vehicle chassis by using revolute kinematic pair. Revolute kinematic pair allows sprung mass to swing relative to the vehicle chassis at the instant centre of the suspension linkage assembly. The swing arm model advantage over the lumped mass model is that it could simulate the change in chamber angle. From the motion point of view the wheels are modeled by using Virtual Tyre Combined Slip Model (fig. 3) developed by Pirelli [Mancosu, 1999]. Each wheel is represent as a single mass and has a spin DOF. Additionally each tire has one vertical, one longitudinal and one axial degree of freedom relative to the rim. Coriolis effects of the spinning wheels are neglected.

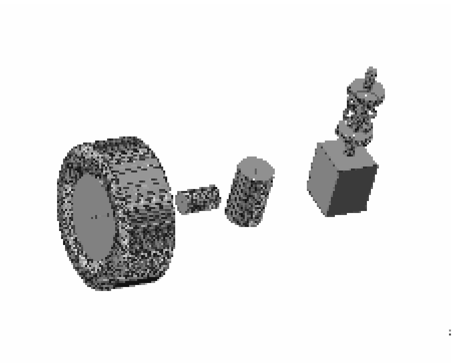

Figure 4. Lumped mass model

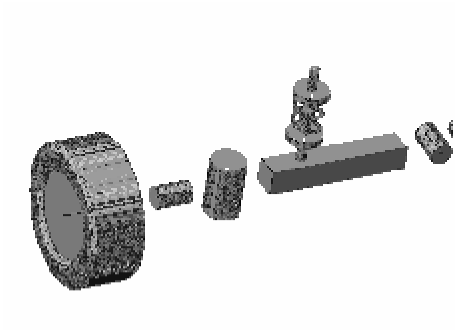

Figure 5. Swing arm model

Six aerodynamic forces and moments are applied to the vehicle chassis at the centre of the mass. Each is quadratic with speed. Coefficients in those equations are quadratic with aerodynamic slip angle. From the force point of view, the suspension elements such as springs and shock absorbers are represented with simple linear model

$$F^d = \begin{cases} k_1 e & e \ge 0 \\ k_2 e & e < 0, \end{cases}$$
$$F^v = \begin{cases} c_1 \dot{e} & \dot{e} \ge 0 \\ c_2 \dot{e} & \dot{e} < 0, \end{cases} \quad (18)$$

or more sophisticated models such as

$$F^d = \left(k_1 Tanh(k_2 e + k_3) - Tanh(k_3)\right)$$
$$F^v = \left(c_1 Tanh(c_2 \dot{e} + c_3) - Tanh(c_3)\right), \quad (19)$$

proposed in the work [Cafferty, 1995]. Vertical tire forces are computed using a linear spring model. If the tire leaves the ground, the force is zero. Tire side forces and aligning moments are computing by using Magic formula [Blundell, 2000]

$$y(x) = D \cdot Sin\left[C \cdot ArcTan\left(B \cdot x - E \cdot \left(B \cdot x - ArcTan(B \cdot x)\right)\right)\right], \quad (20)$$

where

$$Y(X) = y(x) + S_v \text{ and } x = X + S_h. \quad (21)$$

S_v is vertical shift, S_h is horizontal shift, Y is either the tire side force F_z, the aligning moment M_z or the longitudinal force F_x and X is either the slip angle a or the longitudinal slip k .

The steering wheel angle is related kinematicaly to the road wheels by nonlinear lookup table (deg/deg) [Sayers, 1996] that account for linkage geometry (e.g., Ackerman). Wheel steer angle is specified as a function of time. Propulsion forces are calculated by using engine torque and a table lookup involving throttle and engine speed. Tables based on engine speed determine upshifting and downshifting of the transmission. Throttle control is specified as a function of time. A nonlinear table is used to determine brake force for each wheel as a function of hydraulic pressure. Brake force control is specified as a function of time. The road surface is an arbitrary surface, modeled with the help of symbolic spline [Yoshihiko 1998] and measured control points. Surface is specified for each tire as vertical displacements function

$$s_i = s_i\left(\mathsf{G}, \vec{v}, \vec{b}, t\right), \quad i = 1,2,3,4 \tag{22}$$

depend on road surface geometry G, vehicle geometry parameters $\vec{b}$, vehicle velocity $\vec{v}$ and time $t \in [0, t]$.

3.2 Mathematical model and its verification

For proposed vehicle mechanical model a mathematical model has been developed with proposed multibody formalism [Ciglaric, 2001]. The test procedure for the severe lane-change manoeuvre has been used to perform numerical analyisis of the proposed vehicle mechanical model. Manoeuvre is defined in the international standard [ISO3888, 1999]. Test represents a part of the vehicle road-holding ability of passenger cars. Standard specify the dimensions of the test track for duble lane change manoeuvre, which is a dynamic process consisting of rapidly driving a vehicle form its initial lane to another lane parallel to the initial lane, and returning to the initial lane, without exceeding lane boundaries. The driving forces should be constant (the throttle position shall be held as steady as possible). The vehicle Golf I has been used as test subject. The vehicle heading speed was *100km/h*. The time history plot for steering wheel inputs [Blundell 2000] is shown in fig. 6.

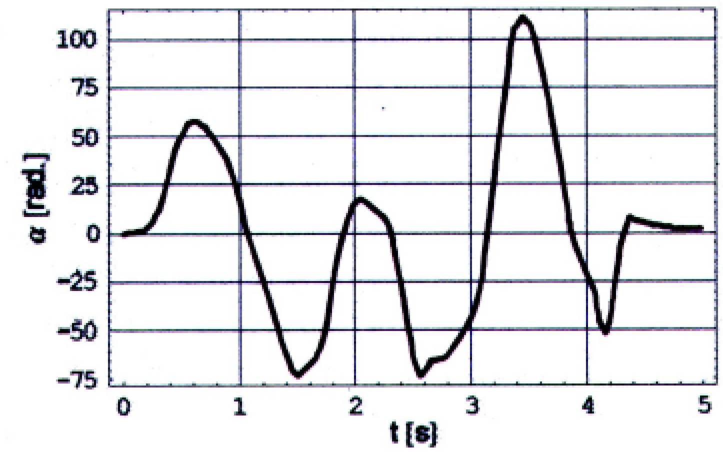

Figure 6. Steering wheel input for the ISO 3888 lane-change maneuver

The study involve the use of Golf I parameter values (the geometry of vehicle chassis, the geometry of vehicle suspensions, the mass and inertia tensors of vehicle chassis, the mass and inertia tensors of vehicle suspensions, the stiffness of suspension springs and tires in vertical direction, the damper characteristics…) as summarize in the work [Ciglaric, 2001]. Also this study involve the use of Golf I tyre test data necessary for Magic formula as available from communication with Professor Heinz Burg from IbB-expetisen.

With the help of proposed vehicle mathematical model, the time history of vehicle position, orientation, velocity, angular velocity, acceleration and angular acceleration has been calculated. Results of calculation done by mathematical models have been presented in virtual environment as shown fig. 7. The virtual environment called *VRMLPath* will be discussed in more detail in the part 2 of this paper.

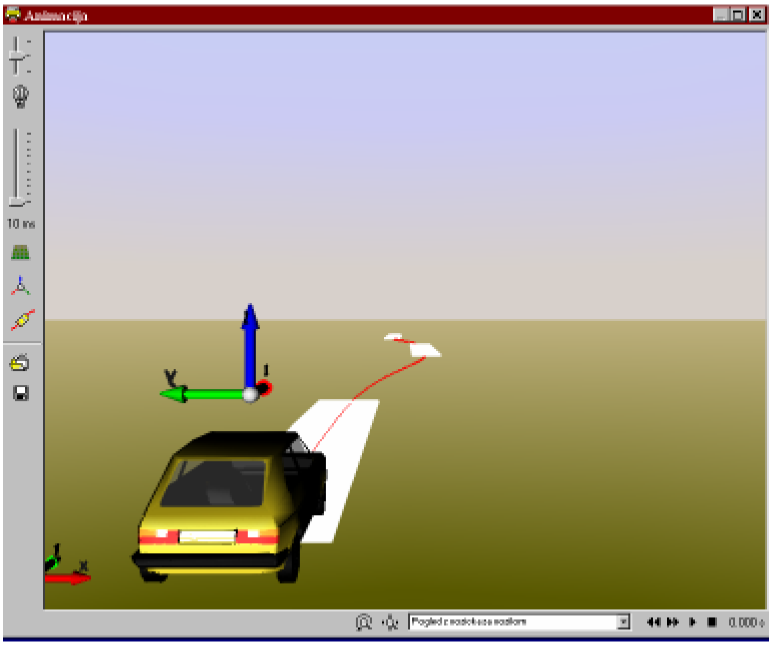

Figure 7. Animation of the lane-change maneuver

The fig. 8 and fig. 9 presents' part of results obtained with simplified mechanical model with 18 DOF and belonging mathematical model. Simplified mechanical model consist of rigid chassis with 6 DOF, four suspensions modeled as lumped mass and simplified VTCSM model, where each tire has only vertical degree of freedom relative to the rim. Each wheel has also a spin DOF. Comparison of calculated and measured dynamic response [Blundell 2000, Ciglaric, 2001] of lateral acceleration and roll angle show good agreement.

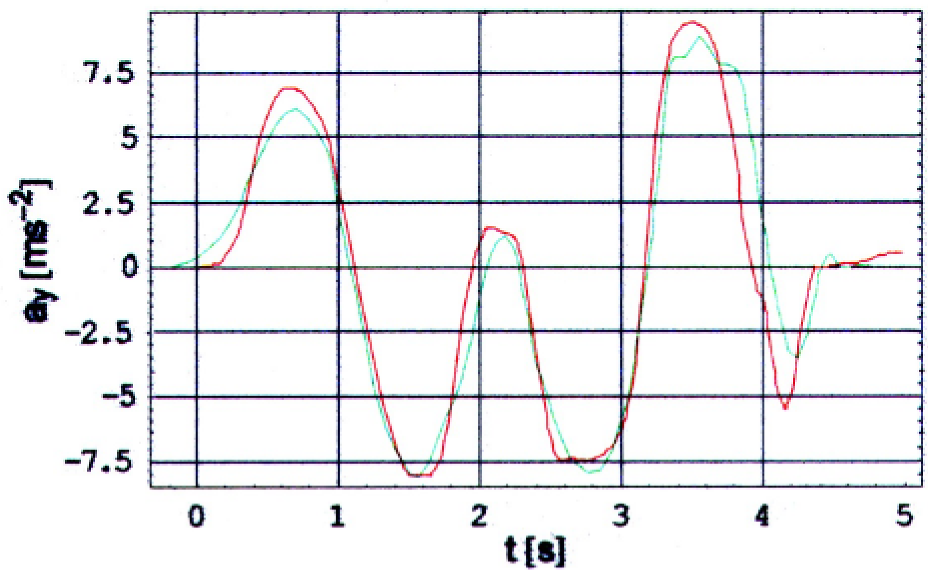

Figure 8. Lateral acceleration comparison (measured in green, calculated in red)

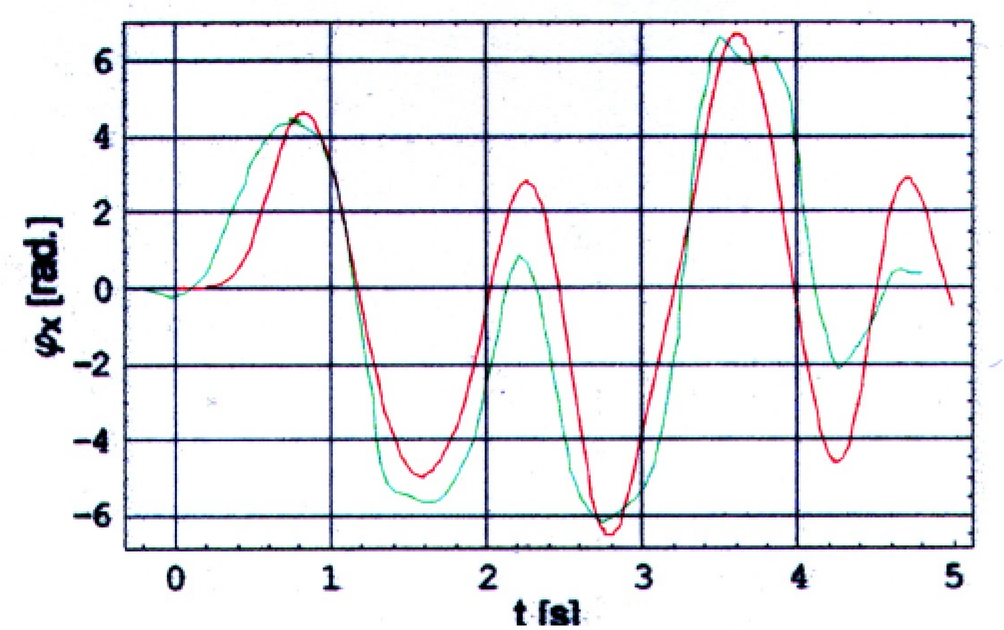

Figure 9. Roll angle comparison (measured in green, calculated in red)

4. Conclusion

The current state of work, presented in this paper, enables quick and easy developing of road vehicle mathematical models in a symbolic form. Numerical analysis of relative simple mathematical model show, good agreement between calculated and measured vehicle dynamic response. A full vehicle model which use simplified representation of suspension system and empirical model Magic formula for tire force representation is suitable for road vehicle pre-impact dynamic analysis. For the road vehicle accidents reconstruction such vehicle models show benefits over sophisticated industrial vehicle dynamic simulation programs. The latest requires many input parameters that are normally not available to road accident reconstruction expert.

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Dr. Iztok Ciglaric
University of Ljubljana
Faculty of Mechanical Engineering
CEMEK laboratory
Aškerceva 6, Ljubljana, Slovenija
Tel.:+386 01 4771 127
Email: iztok.ciglaric@fs.uni-lj.si