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A GRAPH-BASED APPROACH TO CHECK A PRODUCT FUNCTIONAL NET

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1. Introduction

The conceptual design phase involves the design team in analyzing all the possible solutions and selecting the most suitable for a product that satisfies all the requirements from which it originated.

As discussed in literature [Palh and Beitz 1996, Ulrich and Eppinger 2000], an electromechanical device can be treated as a system, in which functional elements interact among them by links. Functional elements can be defined in the classic active verbe-object way, the links are associated to the flow of energy, material and signals that pass through the elements, whereas the link of force describes the connection between the elements. A product can be viewed as a functional net structured as a collection of functional elements and links.

The draft of a suitable functional net allows the designers to put the product, on which they are working, in order. The functional net is also important to check how the product changes if some of its functional elements are modified (change its description adopting new verbe or name) or eliminated. By reasoning at the functional level it is possible to introduce the originality into the design that can lead to product innovation. In order to model univocally a product in term of a suitable functional net and eliminate ambiguity in the definition, a common design language has been proposed [Stone and Wood, 2004].

At different levels, from the high level where the function and the sub-functions are identified, to the lower level, where all components must be defined both morphologically and geometrically, it is important to enhance the nature of the elements and the interrelation among them. As the functional approach can be employed since the first steps of the conceptual phase till the moment when the functions must be performed by a set of components, it is necessary to model the whole design process as a unity. As suggested by the authors [Bruno et al., 2003] the prosess can be implemented in the design space, where the functional blocks are substitute by archetypes, and the links, that connect the blocks in a generic sense, address the functional surfaces of the archetipes where the connection is active.

In order to guarantee that the connection by links is valid at different levels, in the present paper is shown how the graph theory can be employed to check the suitability of a functional net. Such a net is generally complex because many links exist that connect the same functional elements, where the links have different meanings. So the functional net can be treated as a collection of subgraphs, each defined on a proper kind of link. In order to verify the functional net it is necessary to identify each subnet, characterized by the same kind of link, and check each one making use of a set of formal relations that represent the feature of each link. On the basis of these relations a set of controls have been implemented for each subnet.

The applicability of the proposed approach will be tested and discussed on a mechanical device, along the entire design cycle, from the functional level to the embodiment level, where a solution is analysed.

2. Functional nets and the graph theory

A graph G = (N, E) is an abstract object formed by a set of N points (without property) and a set of E edges that connect the points (the sole property of an edge is to connect two points, distinct or the same, without any assumption about the sequence).

There are several ways to enhance the properties of different graphs in the contexts for which they have been designed. Here a few will be recalled, as these are employed in the present approach.

The points are called vertices or nodes, while the edges that connect the nodes in a direct way are called arcs (in this case the graph is said to be directed graph or digraph). If more than one edge joins two nodes, they are considered as multiple edges.

The path of a graph is a sequence of distinct edges between any two nodes.

An edge connecting a node to itself forms a loop. A graph with no loops and multiple edges is a simple graph. A path from a node to itself is called a circuit. If all the nodes are connected via a path, the graph is a connected graph. A subgraph is a subset of nodes and edges of a graph. An unconnected graph may have two or more subgraphs with no connections between them. A simple connected graph with no circuit is a tree. A forest is an unconnected graph in which each subgraph forms a tree.

Graph theory has been applied in several fields of engineering.

In particular in mechanical engineering, the early applications were in the representation of kinematic chains and mechanisms [Dobrianskyi and Freudenstein 1967], from which started a research thread to investigate exhaustively mechanisms and trusses.

Kusiak and Szczerbicki [Kusiak and Szczerbicki 1993] employed a graph representation to evaluate the flow of energy between various components of a design artefact.

More recently [Al-Hakim et al. 2000] the graph theory has been applied to support the conceptual design stage, representing a functional design artefact and idle condition, with a proper visualization of the energy flow, to determine the components of a product that are redundant or cause of loss of functionality in the early design phase.

Shai [Shai 2003] introduced a methodology for solving general engineering problems by transforming engineering knowledge in the dual graph representation, in order to analyse a product and its design process from different points of view.

Depending on the specific survey, vertices and edges can represent the components of an assembly and the relations between them.

In the present approach the vertices of a graph are seen as the functional elements of the functional net that become the components of a product in development phase, and, on the other hand, the edges are associated to the links present in the functional net.

3. Procedures for checking a functional net

A functional net can be seen as a graph formed by the set of all nodes (functional elements or components) and a certain number of edges. As the way the nodes interact is different, the lines can be seen as edges of undirected graphs or as arcs of digraphs. Moreover several multiple arcs can be present, as the interaction between two adjacent nodes is of a different type.

3.1 Definitions

To analyze the relation among the nodes in this kind of structure it is more simple to treat it as a collection of graphs, more precisely the overlapping of four subgraphs, each one acting on a subset of nodes connected by a path of edges or arcs, as shown in Figure 1.

3.1.1 Digraphs

The relations of the functional net associated to the links of energy, signals and material can be analyzed as directed graphs (digraphs). These structures are useful because all these kinds of links are oriented: they follow paths, pass through the nodes.

The digraphs used here are however different at each layer. We represent the digraph associated to the i-th layer with the general notation $\mathbf{D_i} = (\mathbf{N'_i}, \mathbf{E'_i}, \mathbf{i})$, where $\mathbf{N'_i}$ are the nodes, $\mathbf{E'_i}$ the arcs and **i** the weight associated to the arc.

- Energy digraph $\mathbf{D_e}$ = ENERGY = $(\mathbf{N'_e}, \mathbf{E'_e}, \mathbf{e})$ can be structured only as a tree or forest
- Material digraph $\mathbf{D_m}$ = MATERIAL = $(\mathbf{N'_m}, \mathbf{E'_m}, \mathbf{m})$ can be structured only as a tree or forest
- Signal digraph $\mathbf{D_s}$ = SIGNAL = $(\mathbf{N'_s}, \mathbf{E'_s}, \mathbf{s})$ can be structured as a tree or forest, or loop

The number of nodes involved in the schema is generally lower then the total number **N** of nodes.

$$\cup\{ N'_e, N'_m, N'_s \} \subseteq N$$

3.1.2 Undirected graph

Force digraph $\mathbf{G_f}$ = FORCE = $(\mathbf{N'_f}, \mathbf{E'_f})$ can be structured as connected graph.

The number of nodes $\mathbf{N'_f}$ has to coincide with **N** (all the nodes of the functional net).

$$N'_f \equiv N$$

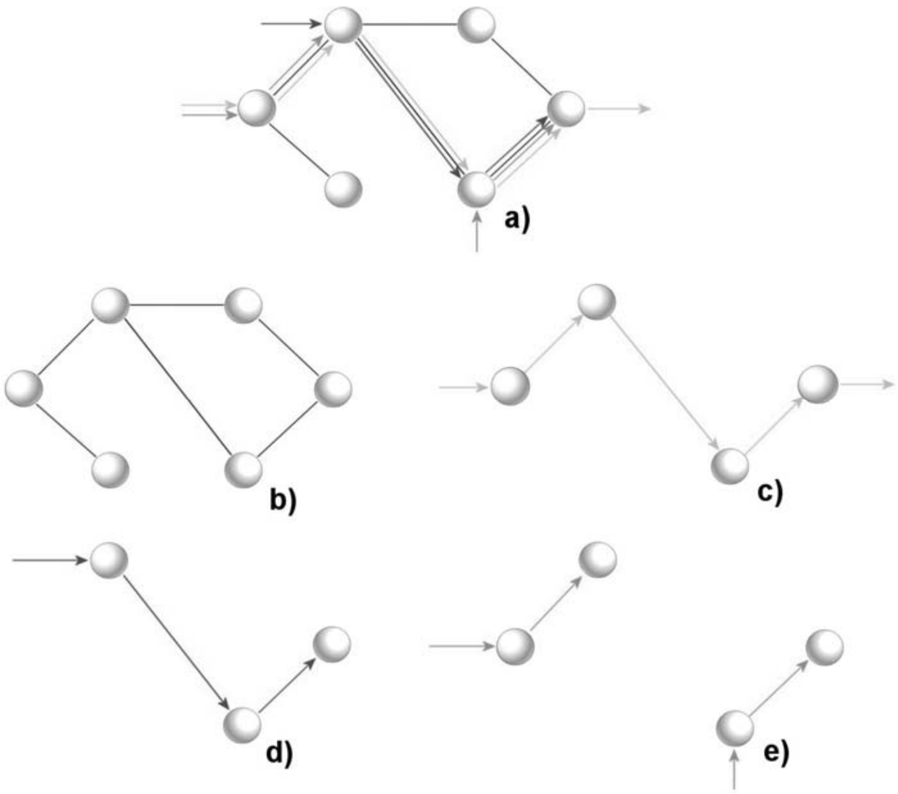

Figure 1. Graph representation of a functional net: a) the whole graph; b) the "FORCE" graph; c) the "MATERIAL" digraph; d) the "ENERGY" digraph; e) the "SIGNAL" forest-digraph

3.2 Check all the links on every layer

As depicted in Figure 1, a functional net can be seen as a collection of graphs. In order to check its validity in representing the work of the design team, a first analysis must involve each sub graph. In the following sections each kind of link, present on a proper layer, is presented and discussed.

3.2.1 Energy layer

This functional link is related to the energy interchange between nodes, though it can have various meanings in different contexts (electric, mechanical, thermal, etc.). An energy link is often present in a device, especially in the conception of an industrial product, because the use of energy, its transformation, or generation/dissipation is required.
The digraphs, which describe this kind of link, are characterized by the presence of open schemes only and this implies the absence of loop structures.
Following the law of energy conservation, which states that energy can be converted from one form to another, but cannot be created or destroyed, it can be said that every functional element (a node) must be characterized by at least one (or more) arcs in input and one (or more) arc in output, unless the node represents a sink or a generator component. In the first case, in fact, we have only an input arc and in the second only an output one.
Following the law of conservation, the energy flow can be represented in a continous mode, either coming from inner nodes or from the outside. In the "ENERGY" digraph the balance of the energy flow has to be "zero" when considering the whole subgraph. Using the concept of "divergence" it is possible to check that this rule is observed.
"Divergence" value, computed at every node **i** of digraph $\mathbf{D_e}$, is defined as the difference between output and input (e_o - e_i), of the energy level associated to the arcs incident to the node. It can have the notation *Div*(i).
In order to check the validity of the energy subgraph, the divergence of the whole subgraph, *Div*(ENERGY), must be evaluated. It can be defined as the sum of *Div*(i), computed on all the nodes $\mathbf{N'_e}$ of $\mathbf{D_e}$:

$$\Sigma\, Div(\mathrm{i}) = Div(\mathrm{ENERGY}) \Rightarrow Div(\mathrm{ENERGY}) = 0 \qquad (1)$$

If this is not satisfied the designer has to review the subgraph and try to identify the problem.

3.2.2 Material layer

The material layer follows another natural law of conservation along the paths, where the flow is not dispersed but remains the same, although the state of the material can change. Nodes can be considered "active" if they carry material through the path. If a material link exists there will be at least one path with one arc from node i to node j, characterized by a connection of continuity.
To check a flow on the material digraph, MATERIAL = ($\mathbf{N'_m}$ **,** $\mathbf{E'_m}$, **m**), it is necessary to make a check on each path and on each node of the path.
Letting i and j be two nodes of the material digraph, the principle of mass conservation is verified if the following conditions are satisfied:

1. a path P_{ij} ($P_{ij} \in \mathbf{E'_m}$) from i to j formed by a set of arcs (at least one arc) exists
2. at every node of the path the balance relation,

$$\Sigma F_{in} = \Sigma F_{out} \qquad (2)$$

is valid, where ΣF_{in} is the sum of the input flow in the i-th node, and ΣF_{out} the sum of the flow in output.
The two conditions are applied to all the "inner" nodes of all possible paths of the material digraph. The "boundary" nodes, both initial and final, are processed by the designer to check the functional meaning of the node, so that the material link can originate outside the net or can terminate in a storage component.

3.2.3 Signal layer

Signal flow can be treated as digraphs to describe complex systems composed of several linked parts. This is a valid alternative with respect to the classical model of "block diagrams", already popular due to their simple manipulation. For example a feedback loop, that shows a signal cycle, can be drawn as an equivalent flow digraph. The signal digraph SIGNAL = ($\mathbf{N_s', E'_s, s}$) can be formed by a collection of paths, even trees and loops, that can also be disconnected.

In these digraphs the nodes can be classified as *independents* (or *sources*) which do not enter any arc, and *dependants,*in which there are at least one arc. The arcs can be identified by a value.

Considering the oriented nature of the signal, note that the input signal in a node can be transfer in output on several arcs:

1. by activating one arc at a time mutually (switching); or
2. by activating more than one arc at a time, duplicating the total signal level.

In order to give an accurate representation two checks are made on the digraph:

1. external (in case of activation or survey) with the check on the path, controlling the interrelation among nodes and arcs;
2. internal (in case of regulation) adding to the previous an additional further monitoring to the feedback loops.

3.2.4 Force layer

The force layer contains all the information related to the links between couples of components (i.e. contacts and positioning). Unlike all previous layers, the force does not have a verse, being addressed at indicating the mutual interaction. Therefore it can be modelled as a pure graph.

In the approach, the nodes are joined by simple edges, without arrows, and the schema must be connected. In this situation the structure can be *weakly connected,* and the number of edges are N-1, or *strongly connected* and the number of egdes greater than N-1.

To make this reasoning viable, it is necessary that the "FORCE" subgraph has an important characteristic: it must be "connected", that is, a node cannot be isolated. Moreover the situation where a node is only connected by one single edge, or when its cardinality (the number of incident edges) is greater than 3 must be clarified. This helps the designer, who reflects on the functional role of a component in the analysis phase, to solve the problems relating to the solution required.

During the preparation of the "FORCE" graph, if different numbers of loops are present, the designer can verify the possibility of combining joined nodes (for every loop structure) in a more complex component.

In substance, the "force" graph gives a good approximation of the preliminary architecture of the entire product in the conceptual phase.

4. Application

The construction of a functional net can be the first step in design activity, when in a creative way designers try to solve a problem for the first time, or in redesign, when a new solution is required. The procedures proposed in the previous sections will be applied to a simple mechanical device, already discussed by means of the graph theory [Al-Hakim, 2000], so a functional net can be extracted. The net will be discussed in term of subgraphs and several considerations will be made.

The test case is related to the established problem of "change angular speed" (see Figure 2). This first high-level functional task involve, in a general sense: a certain level of energy in input; its modified value in terms of angular speed or torque in output; a signal in input to activate the process.

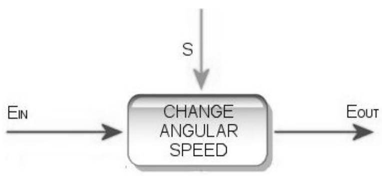

Figure 2. Functional net for the high level task

Starting from this, a more detailed structure can be obtained if the initial function is decomposed into more simple sub-functions. In Figure 3 a functional net is reported, where the functional analysis is done for a solution that allows to obtain two speed ratios. A set of functional elements are present, connected in different way.
The whole system is contained in a housing with the task "contain the device". The elements "input the energy" and "output the energy" are connected with the outside, and respectively the first inputs the energy to be transformed and the second outputs the energy transformed. Other two elements allow to "reduce" or "increase the angular speed". These latter elements can be selected one at the time, so a further element must be present with the task "actuate the ratio".

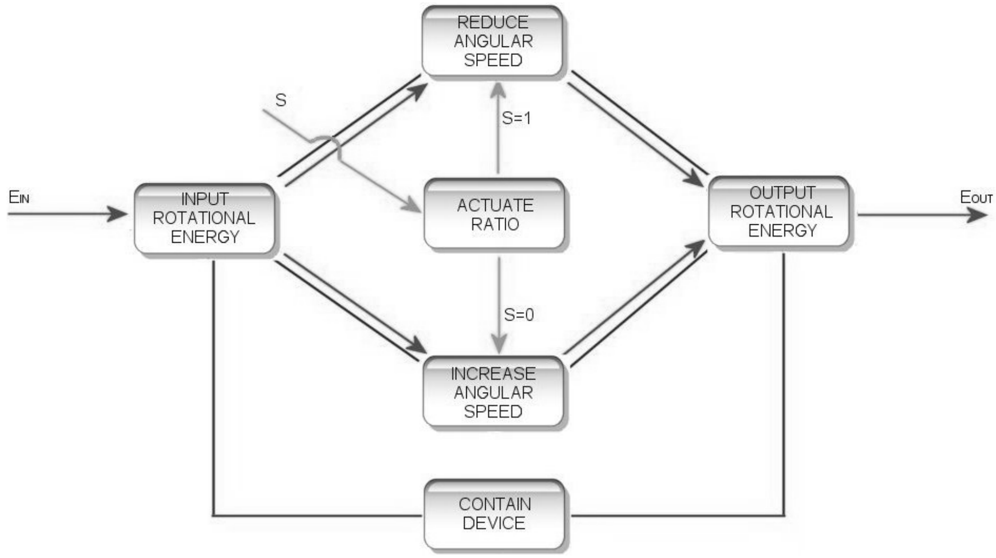

Figure 3. Functional net of a partially defined solution

Even if Figure 3 seems be sufficiently clear to depict the problem under investigation, it contains a set of inconsistencies: 1) Energy can not simultaneously flow through both gears; 2) The element that "actuate the ratio" can not be isulated, it must be linked to something else.
In order that the construction of a functional net can be really the first step of the design process the designers should see this approach as a real tool to stimulate their creativity. The checks of the net proposed in the paragraph 3.2, made by graph theory, suggests to review the ENERGY digraph, because the condition (1) that control the flow of energy is not satisfied on the node "input the energy", because the energy seems forked in two parts. Furthermore, the check of the FORCE graph enhance that a functional element "actuate the ratio" is not connected to the net. But really, for this problem, the analysis performed at the linguistic level of the functional reasoning can not add any further element. It is necessary to concretize the reasoning towards a real solution, maintaining the functional point of view.
So the design team is urged to research a valid solution, choosing the most suitable way to connect the functional element "actuate the ratio". In Figure 4.a a classic clutch drive system is drawn, where the actuator is linked to the output element (shaft). Briefly, from the input shaft A the energy flows to output shaft B by means of two pairs of gears (D and H or F and L) which are permanently in contact. The gears D and F are keyed to the input shaft. Gears H and L slip on the output shaft and are not keyed on it. A double coupling I is keyed to the output shaft and slides on it. Only one pair of gears, D and H or alternatively F and L, transmits torque to the output shaft. The latter depends on the position of the coupling. All the components are contained in the casing C. In figure 4b a functional net is reported which outlines the flows of energy and signal and the contact of force among the elements, in the case when the clutch is idle, but the gears H and L are in rotation.

If a signal (of exclusive OR) arrives at element I, one of the two speed ratios can be activated (see figure 4c). If the coupling I is jointed to H the energy flows to B following the path A-E-D-H-I-B, and angular speed and torque change values according to the ratio produced by the gears D and H. In any case a dead-end track of energy remains from A to L. A similar situation exists when I is jointed to L. In figures 4d, 4e and 4f the subgraphs involved (energy, force and signal) are reported.

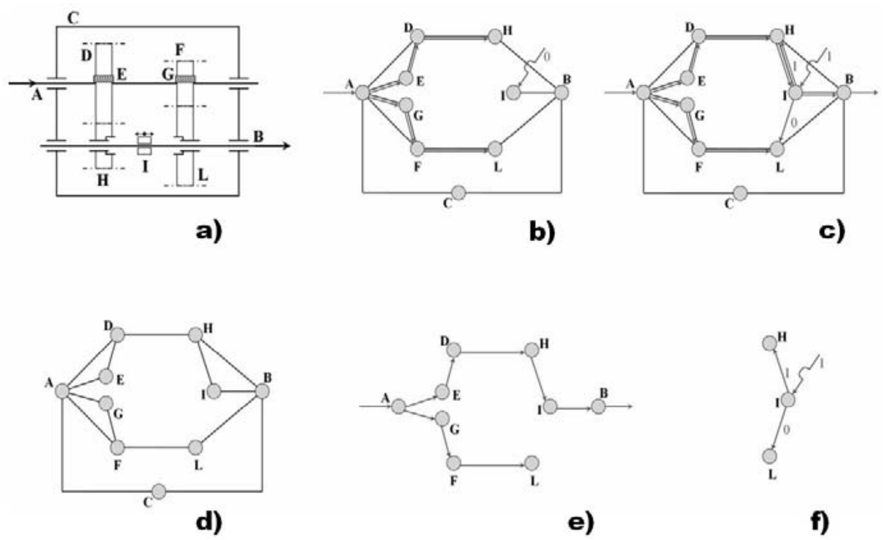

Figure 4. Functional analysis of a clutch drive system: a) draft; b) functional net in the idle configuration; c) functional net in operating condition; d) the "FORCE" graph; e) the "ENERGY" digraph; f) the "SIGNAL" digraph.

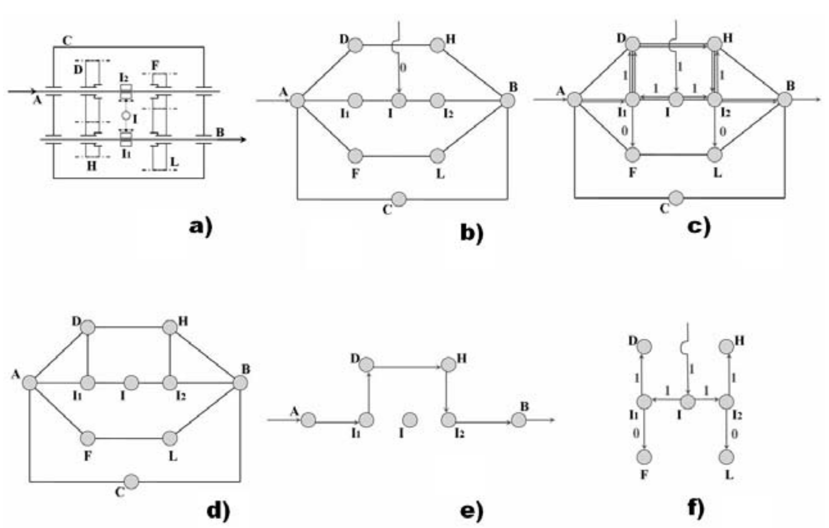

Figure 5. Functional analysis of a double-clutch drive system: a) draft; b) functional net in the idle configuration; c) functional net in operating condition; d) the "FORCE" graph; e) the "ENERGY" digraph; f) the "SIGNAL" digraph.

However, the procedure able to check the energy subgraph runs in an exception, because a path exists in which it recognizes the element L as a dissipator of energy. So designers have to decide if this situation is acceptable or not. So the attempt to eliminate this idle path can lead to a new configuration for the device. For example, also by allowing gears D and F to slip freely on the shaft A, a new functional net can be drafted, as shown in figure 5c. In order to guarantee the energy flow a more complex clutch system should be devised (see figure 5a), activated by a suitable signal, that simultaneously couples shaft A with gear D (or F) and shaft B with gear H (or L). So the energy flows through the elements A-I1-D-H-I2-B and the path A-F-L-B is unloaded.

5. A Proposal for a Doubt Based System, conclusions and future work

The basis of the methodology proposed is a conception of the design process as something that aims to always keep the designers at the center of this activity. In fact, they have to control the correctness of the functional net continuously. In order to facilitate the design work, a set of guidelines have been formalized that control the suitability of the links introduced in the net.

The net can be checked continuously, so the design work is constantly questioned, sowing doubts in the designers's mind about the correctness of the layout under conception. In this way the designers are able to identify errors immediately and they can eliminate them.

This aspect is fundamental to the design process, particularly, considering that the main mistakes derive from carelessness, or worse from poor knowledge of the interaction among the functional elements of a product. In fact, inconsistent solutions are very frequent because of illogical links established between different elements of the net.

The employment of the graph theory in this context appears valid. The mathematical formulation of the relations among all the elements involved, though it is located at a high level of abstraction, makes it possible to work with this set of rules in a computer logic context. In the near future a set of procedures will be formalized and a tool environment will be devised.

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